Aspects of Non-Associative and Non-Commutative Geometries in String Theory

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General Information

A theory of quantum gravity in a non-commutative space-time is expected to be renormalizable, because of the grained structure of the space-time. However, from a string theory point of view such a theory cannot be constructed. This is because the graviton is in the spectrum of closed strings, however, non-commutative geometries are observed in the open string sector in D-brane physics.

Successive T-dualities in the closed string sector is very instructive about the hidden aspects of the string theory. Three successive T-dualities performed in the closed string sector in flat space with a constant three-form flux leads to a theory with so called R-flux background, which is not an ordinary geometry even locally. This is the non-associative geometry. One might try to find asymmetric CFTs which would correspond to non-associative target space geometries. One such attempt is done by starting with SU(2) WZW model with H-flux background. The corresponding R-flux background have properties of a non-associative target-space structure which is shown to be described in terms of a deformed tri-product.

We plan to analyze the effect of triple T-duality transformation on the string spectrum on AdS_3 and analyze the CFT obtained after these transformations. That CFT is expected to flow to an asymmetric CFT, which has to be showed and analyzed. From the scattering amplitudes of vertex operators we plan to derive a tri-product which would describe the non-associative target space structure. Form of tri-product on a Lorentzian manifold is important, because it is needed if one tries to define a non-associative gravity on a Lorentzian non-associative space-time. If one considers WZW model based on the non-compact supergroup SU(2/1,1), the central charge of the bosonic sector becomes equal to 6, independent of the Kac-Moody level, k. The bosonic part of this supergroup is SU(2)xSL(2,R) and corresponding bosonic manifold is S^3 x AdS_3. In the $k \to \infty$ limit, AdS_3 tends to flat 3-dimensional Minkowski space while the rest of the theory adjusts to this limit smoothly. Having a tri-product, one could define a non-associative gravity theory in the corresponding R-flux background. After the appropriate $k \to \infty$ limit one can try to compactify and search the 4-dimensional ramifications of such a theory. To compare the predictions of non-associative gravity with the predictions of general relativity one needs to map the non-associative theory to a associative one a la Seiberg-Witten. That could be / should be done by equating the gauge orbits of two theories as in the original construction for non-commutative theories.

Non-associative geometries could appear not just in closed string sector of string theory after multiple T-dualizations. It is possible that they also appear in M-theory. Theories for multiple M-branes contain non-associative three-algebras in their construction implicitly. This means that they have the necessary ingredient of a constant 3-form flux naturally. With T-dualizations one could again reach to a non-associative R-flux geometry. The analysis of that R-flux background will contain its own difficulties because the corresponding theory is not a two-dimensional CFT.

Besides non-associative structures, associative non-commutative also naturally emerge within this same context of open strings and D-branes. Moreover, one may look at this rather formally, formulating in pure algebraic terms. For instance, in string theory the multiplication of fields is associated with the sewing operation and with pant diagrams. This induces a multiplication operation which satisfies various constraints. The principal difference between the open and closed sectors is that in the former case the fields carry additional indices from the set of "boundary conditions" (or "D-branes"). Pure algebraic formulation of string theory appeals to its topological structure. Hence, a necessity of study simpler examples of topological theories. We plan to study two such examples: the Hurwitz theory and the Chern-Simons theory.

In Hurwitz theory the closed-string algebra is that of the Young diagrams (conjugation classes of permutations). This implies that the open-string fields will be labeled by pairs of Young diagrams with some additional data. One can identify them with bipartite graphs: conjugation classes of pairs of permutations.

In Chern-Simons theory the basic objects to calculate are the Wilson averages, the most non-trivial ones are those associated with knotted contours. They coincide with knot polynomials (HOMFLY polynomials or their specializations). This naturally leads to studies in knot theory.

In fact, many more simple examples of the structures described above are provided by matrix models. We plan to investigate various types of matrix models suitable for describing different contexts, from Chern-Simons theory to T-duality and M-theory.

Key Words: T-duality, asymmetric CFT, R-flux background, non-associative geometry, tri-product, WZW model, non-associative gravity, Seiberg-Witten construction, multiple M-branes, topological strings, Chern-Simons theory, matrix models, Hurwitz theory.